

1. A disease occurs in 3% of a population.

(a) State any assumptions that are required to model the number of people with the disease in a random sample of size n as a binomial distribution. (2)

(b) Using this model, find the probability of exactly 2 people having the disease in a random sample of 10 people. (3)

(c) Find the mean and variance of the number of people with the disease in a random sample of 100 people. (2)

A doctor tests a random sample of 100 patients for the disease. He decides to offer all patients a vaccination to protect them from the disease if more than 5 of the sample have the disease.

(d) Using a suitable approximation, find the probability that the doctor will offer all patients a vaccination. (3)



5. A continuous random variable X has the probability density function $f(x)$ shown in Figure 1.

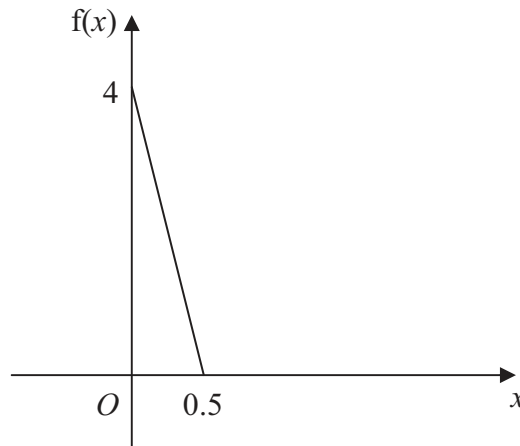


Figure 1

- (a) Show that $f(x) = 4 - 8x$ for $0 \leq x \leq 0.5$ and specify $f(x)$ for all real values of x . (4)

- (b) Find the cumulative distribution function $F(x)$. (4)

- (c) Find the median of X . (3)

- (d) Write down the mode of X . (1)

- (e) State, with a reason, the skewness of X . (1)



6. Cars arrive at a motorway toll booth at an average rate of 150 per hour.
- (a) Suggest a suitable distribution to model the number of cars arriving at the toll booth, X , per minute. **(2)**
- (b) State clearly any assumptions you have made by suggesting this model. **(2)**

Using your model,

- (c) find the probability that in any given minute
- (i) no cars arrive,
- (ii) more than 3 cars arrive. **(3)**
- (d) In any given 4 minute period, find m such that $P(X > m) = 0.0487$ **(3)**
- (e) Using a suitable approximation find the probability that fewer than 15 cars arrive in any given 10 minute period. **(6)**



7. The queuing time in minutes, X , of a customer at a post office is modelled by the probability density function

$$f(x) = \begin{cases} kx(81 - x^2) & 0 \leq x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{4}{6561}$. (3)

Using integration, find

(b) the mean queuing time of a customer, (4)

(c) the probability that a customer will queue for more than 5 minutes. (3)

Three independent customers shop at the post office.

(d) Find the probability that at least 2 of the customers queue for more than 5 minutes. (3)

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